More problems on Power Sums and Bernoulli Numbers

Some participants say that some of the problems introduced are well known (I suppose that fans of «Concrete Mathematics» comment in this way on all problems which deal with formulae). If some problems are known to you, you may just discuss the solutions with the members of the jury responsible for the problem (Volodya Dotsenko, Vadim Bugaenko) — you don't need to write down solutions in this case.

Here you see two more problems — they were introduced during the presentation.

17. (Kummer's congruence) Prove that for integers n, k and prime p such that $2n$ is not divisible by $p-1$ and $2n-2k$ is divisible by $p-1$ the numerator of $B_{2n}/2n - B_{2k}/2k$ is divisible by p.(And so you should check only finitely many cases in order to find out if given p is regular.)

18. Prove that there are infinitely many irregular primes. (Hint. You can use the following fact: $|B_{2k}/2k| > 1$ for large k.)