The Hunting on the Snark

The Baker hunts on the Snark on infinite squared paper. In one move the Snark can jump from square (x, y) to any square (x', y') such that $|x - x'| \leqslant n$, $|y - y'| \leqslant n$. The number n is called *velocity* of the Snark. The Baker's move is placing a cake on any empty square. The Snark hates cakes, so he never jumps to a square with a cake. The Baker's goal is to lock the Snark in such a way that the Snark cannot jump. The Snark's goal is opposite, i. e., not to let the Baker lock him.

0. Prove that the Baker can lock the Snark of velocity 1.

1^{**}. **(Open problem)** For at least one $n > 1$ find out who can reach his goal: the Baker or the Snark.

We propose several easier questions about somewhat restricted Snarks.

ε. Suppose that with each move the Snark increases his y-coordinate ($y' > y$). Prove that the Baker can lock him.

2ε. Suppose that the Snark never decreases his y-coordinate ($y' \geqslant y$). Prove that the Baker can lock him.

3ε. Suppose that once the Snark reaches square (x, y) he never visits squares with y-coordinate less than $y - 1\,000\,000$. Prove that the Baker can lock him.

4ε. Suppose that with each move the Snark increases his distance from the origin. (The distance between the origin and square (x, y) is defined as $\max\{|x|, |y|\}$.) Prove that the Baker can lock him.

5ε. Suppose that once the Snark reaches a square at distance d from the origin he never visits squares located at distance less than or equal to $d - 1\,000\,000$. Prove that the Baker can lock him.

1/2001. Suppose that the Snark is suspicious in the following way: if once he could have ever jumped to a square but he didn't, then he will never jump to this square. Prove that if the Baker can lock the suspicious Snark then he can lock any Snark of the same velocity.

Now the hunting takes place in three-dimensional space divided into cubes.

δ. Suppose that with each move the Snark can jump to any cube having a common face with his current location (i. e., he has six potential moves). Prove that the Baker can lock him.

$1 - \delta$. Suppose that with each move the Snark can jump from a cube (x, y, z) to any cube (x', y', z') where $|x - x'| \leqslant 1$, $|y - y'| \leqslant 1$, $|z - z'| \leqslant 1$ (i. e., he has 26 potential moves). Can the Baker lock the Snark?