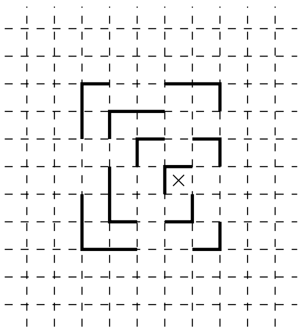
The numbers in parentheses given after the numbers of the problems indicate grades of Russian school. The 11th grade in Russian school is the last year before graduation. Solution of the problems meant for senior grades is welcome. The problems for junior grades do not affect the final score.

1. (6–7) Two merchants sell plums. In the plums sold by first of them, the stone occupies one third of the weight. In the plums sold by the second one, the weight of the stone is half of the total weight of the plum. But the first merchant sells a kilogram of his plums for 150 roubles, and the second merchant sells a kilo of his plums for 100 roubles. Whose plums will an intelligent customer buy, and why?



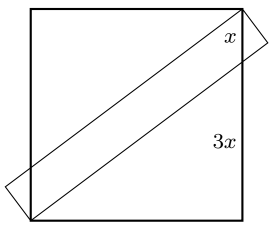
2. (6–8) A tiger is locked inside a fence, formed by a non-self-intersecting closed polygonal chain, edges of which go along the lines of the square grid. A part of this fence is shown on the figure (the position of the tiger is marked by a cross). Draw a possible form of the whole fence.

3. (6–11) A witch has two sandglasses: one for 2 minutes and another for 5 minutes. Brewing her potion, she has to boil it for exactly 8 minutes. Unfortunately, when the boiling started, she noticed that some part of the sand in the first (2-minute) sandglass is still in the top half. However, all the sand in the second (5-minute) sandglass is in the bottom half. How can she determine the moment to take the cauldron off the fire, measuring exactly 8 minutes? (The sand falls with a constant speed, and you can neglect the time needed to reverse the sandglass.)

4. (8–9) In the product $a^n b^m c^k$, the bases and the degrees should be positive integers. First, Tweedledum specifies numerical values for three letters. After that, Tweedledee specifies the values for the remaining letters, attempting to do it so that the resulting product is a cube of a positive integer. Can he always reach his goal, or can Tweedledum choose his numbers to prevent him from doing so?

5. (8–11) A convex quadrilateral is drawn on a blackboard. Three boys made one claim each: Alexey said, “This quadrilateral can be cut by its diagonal into two acute triangles”. Boris replied: “This quadrilateral can be cut by its diagonal into two right triangles”. And Charlie concluded: “This quadrilateral can be cut by its diagonal into two obtuse triangles”.

It turned out that just one of them was wrong. Name the boy who certainly was right, and prove that he was.



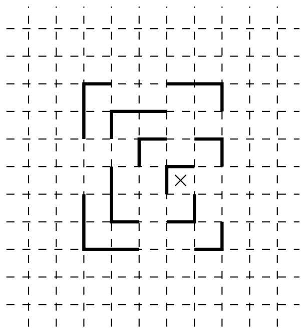
6. (10–11) A side of a rectangle of area 14 divides a side of a square at the ratio $1:3$ (see the figure). Find the area of the square.

7. (10–11) Positive integers m and n are such that the sum $\sqrt{n}+\sqrt[3]{m}$ is also an integer. Is it necessarily true that both summands are integers?

XXXIV Lomonosov Tournament 25 September 2011
Mathematics Competition

The numbers in parentheses given after the numbers of the problems indicate grades of Russian school. The 11th grade in Russian school is the last year before graduation. Solution of the problems meant for senior grades is welcome. The problems for junior grades do not affect the final score.

1. (6–7) Two merchants sell plums. In the plums sold by first of them, the stone occupies one third of the weight. In the plums sold by the second one, the weight of the stone is half of the total weight of the plum. But the first merchant sells a kilogram of his plums for 150 roubles, and the second merchant sells a kilo of his plums for 100 roubles. Whose plums will an intelligent customer buy, and why?



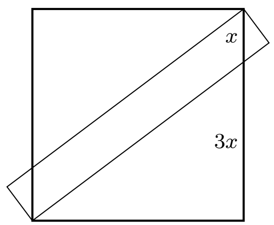
2. (6–8) A tiger is locked inside a fence, formed by a non-self-intersecting closed polygonal chain, edges of which go along the lines of the square grid. A part of this fence is shown on the figure (the position of the tiger is marked by a cross). Draw a possible form of the whole fence.

3. (6–11) A witch has two sandglasses: one for 2 minutes and another for 5 minutes. Brewing her potion, she has to boil it for exactly 8 minutes. Unfortunately, when the boiling started, she noticed that some part of the sand in the first (2-minute) sandglass is still in the top half. However, all the sand in the second (5-minute) sandglass is in the bottom half. How can she determine the moment to take the cauldron off the fire, measuring exactly 8 minutes? (The sand falls with a constant speed, and you can neglect the time needed to reverse the sandglass.)

4. (8–9) In the product $a^n b^m c^k$, the bases and the degrees should be positive integers. First, Tweedledum specifies numerical values for three letters. After that, Tweedledee specifies the values for the remaining letters, attempting to do it so that the resulting product is a cube of a positive integer. Can he always reach his goal, or can Tweedledum choose his numbers to prevent him from doing so?

5. (8–11) A convex quadrilateral is drawn on a blackboard. Three boys made one claim each: Alexey said, “This quadrilateral can be cut by its diagonal into two acute triangles”. Boris replied: “This quadrilateral can be cut by its diagonal into two right triangles”. And Charlie concluded: “This quadrilateral can be cut by its diagonal into two obtuse triangles”.

It turned out that just one of them was wrong. Name the boy who certainly was right, and prove that he was.



6. (10–11) A side of a rectangle of area 14 divides a side of a square at the ratio $1:3$ (see the figure). Find the area of the square.

7. (10–11) Positive integers m and n are such that the sum $\sqrt{n}+\sqrt[3]{m}$ is also an integer. Is it necessarily true that both summands are integers?