

Mathematical Game Competition

You need to choose one of the three games below (the most interesting for you) and to come up with a winning strategy for the 1-st or the 2-nd player. Your strategy must guarantee a victory regardless of the opponent's moves. Try not only to describe your move but also to explain why the victory is inevitable. A solution without proper explanation doesn't count.

Do not rush into solving all the tasks as you need to save time and energy for other competitions. A good analysis of even a single game would be considered as a success.

(Results of the mathematical game contest do not affect possible benefits in high school admissions which could be granted to the winners and awardees of the Lomonosov Tournament 2011.)

1. "Linear tic-tac-toe". In this game an array (a strip) of N cells is the playing field. There are two players. The 1-st player puts two crosses (X) in any two empty cells. The 2-nd player puts a single nought (O) in any empty cell. The 1-st player wins if there are five crosses (X) in a row. And the 2-nd player wins if he can prevent this from happening.

Who wins the game (the 1-st or the 2-nd player) regardless of the opponent's turns? Consider the following cases:

- a) $N = 10$
- b) $N = 13$

Also consider a variant of the game when the ends of the strip are joint together to form a loop (a ring). Who wins in the cases when:

- c) $N = 8$
- d) $N = 9$
- e) $N = 12$

2. "Staking rings". There are two players. They have five sticks and an unlimited number of rings. On every turn a player puts rings on four sticks (one on each he has chosen). Initially there are no rings on sticks. A player loses when he cannot make his next turn.

Who wins (the 1-st player or the 2-nd player) regardless of the opponent's turns in the cases when:

- a) the maximal number of rings on a stick is 8
- b) the maximal number of rings on a stick is 7
- c) the maximal number of rings on a stick is 5 but on every turn a player puts rings on 3 sticks (one on each)
- d) there are 100 sticks, the maximal number of rings on a stick is 95, and on every turn a player puts rings on 98 sticks (one on each)

3. "Don't lose a balloon!" Several balloons are linked together with heavy strings as shown on the picture below. There are two players. On every turn a player untie a single string and removes it. If after a player's turn some balloon isn't connected to the strings any more then the balloon flies away and the player loses the game.

Who wins (the 1-st player or the 2-nd player) regardless of the opponent's turns?

Consider the cases described on the following pictures (circles represent balloons and bold lines represent strings).

