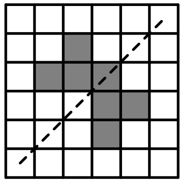
Mathematics Competition

The numbers in parentheses given after the numbers of the problems indicate grades of Russian school. The 11th grade in Russian school is the last year before graduation. Solution of the problems meant for senior grades is welcome. The problems for junior grades do not affect the final score.

1. (6–7) Monkey, Donkey, and Goat decided to play some music. They sat down on three chairs in a row, Monkey taking the right seat. They fiddled away for some time but made nothing but noise.

Trying to improve their music the animals changed places. Now Donkey ended up in the middle. Seeing no improvement they changed seats again. It turned out that each animal sat on every chair (left, right and middle) once. Find the last position of each animal.

2. (6–8) Vanya colored in several squares on a quad-lined piece of paper. The shape that Vanya got had no lines of symmetry. Next, Vanya colored one more square and claimed that the new shape had 4 lines of symmetry. Can Vanya possibly be correct? (Example: the shape on the right has one line of symmetry shown by the dotted line.)



3. (6–8) Some students in Mr. Thompson's class like to watch soccer, and some like watching cartoons. There are no students that don't watch either of the two. The average grade of cartoon-watchers is less than 3. The soccer-watchers' average grade is less than 3 as well.

Is it possible that an average grade of the whole class is greater than 3? (An average (or mean) of several grades is equal to the sum of these grades divided by their number. The grades can be 0, 1, 2, 3, 4.)

4. (7–11) The talking scales tell you the weight of an item rounded to the nearest integer number of kilograms. Vasya used these scales three times to weigh himself and several identical bottles with water, and recorded the results of the weighings in this table:

	Vasya and 5 bottles	Vasya and 10 bottles	Vasya and 14 bottles
The talking scales said:	"22 kilograms"	"25 kilograms"	"28 kilograms"

Can this table be correct? If you think that this is possible, give a fitting example of how much Vasya and the bottles could weigh.

(When a fractional part of a number is less than 0.5, the number is rounded down. When it is 0.5 or greater, it is rounded up. For example, 3.5 is rounded to 4.0.)

5. (8–11) An isosceles triangle with a 120° angle is made out of 3 layers of folded paper. When it was unfolded, we got a rectangular piece. Draw such a rectangle and plot the folding lines on it.

6. (9–11) Each cell of a 7×7 square contains a number. The sum of numbers in every 2×2 square and in every 3×3 square is equal to 0. Prove that the sum of numbers in 24 perimeter cells of the 7×7 square is equal to 0 as well.

7. (9–10) Given an arbitrary triangle, is it always possible to place three positive numbers at its vertices such that the length of each side of the triangle is equal to the sum of the numbers at the end-vertices of that side?

8. (11) Prove that for any tetrahedron it is possible to place a nonnegative number on each of its edges in such way that an area of every face of the tetrahedron is equal to the sum of the numbers on the surrounding edges.