Mathematics Competition

For each problem, the recommended grades are specified in the parentheses in front of the problem. A student is allowed to solve problems intended for older grades. The problems for younger grades do not count for the results. (The 11th grade in Russian school is the last year before graduation.)

1. (6–7) Masha has 2 types of coins: 5-rouble and 2-rouble ones. With this money, she is planning to buy several pies. If she pays with all her 2-rouble coins, she will need extra 60 roubles to buy 4 pies. If she pays with all her 5-rouble coins, she will need extra 60 roubles to buy 5 pies. If she combines all her 2-rouble and 5-rouble coins together, she will need extra 60 roubles to buy 6 pies. What is the price of a single pie?

2. (6–8) Some geometric figures can be tiled with dominoes; for example, a 6 by 6 square can be tiled with dominoes. Some figures cannot: for example, a 5 by 5 square cannot be tiled. (Dominoes are 2 by 1 and 1 by 2 rectangles. "To tile" means to cover completely and without overlaps.)

Come up with a figure such that

— This figure cannot be tiled with dominoes.

— A new figure that is formed by adding a single domino tile to the original figure can be tiled with dominoes.

To submit the solution, draw this figure on grid paper. The lines of the figure should follow the grid lines; if cut out, the figure should not fall apart into 2 or more pieces. Draw the domino tile that you are adding. (Shade it.) Draw the tiling of the new figure.

3. (6–11) 36 sumo wrestlers came to the tournament. Each wrestler has a rating. Whenever two wrestlers with different rating are having a match, the fighter with the higher rating wins. A match of two wrestlers with the same rating always ends in a draw.

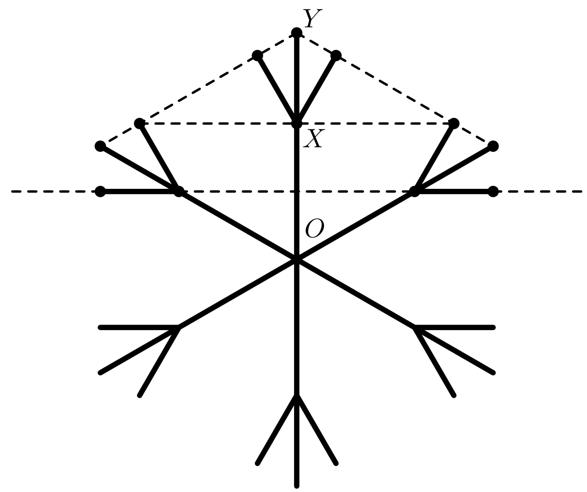
Find out whether the following statement is correct:

"It is always possible to split these 36 wrestlers into pairs in such a way that

— A winner in each pair will be at least as strong as each of the losers (or those who had a draw) in all 18 pairs, and

— Everyone who has a draw will be at least as strong as each of the losers in all 18 pairs".

4. (8–10) The snowflake on the figure below has rotational symmetry: if rotated 60° around point O, the snowflake will match itself. It also has reflectional symmetry with respect to the line OX. Find $OX : XY$ — the ratio of the lengths of the segments OX and XY. (Dotted lines connect points that belong to the same lines.)



5. (8–11) The straight-A student Boris knows the right way to add fractions. The lazy student Peter adds fractions in a different way: the sum of the denominators becomes the new denominator, the sum of the numerators becomes the new numerator.

The teacher asks each of the boys to add three fractions, each reduced to a simplest form. Boris's answer is correct: the sum is equal to 1. Can it be possible that Peter's answer is less than $\frac{1}{10}$?

6. (10–11) The "Young Geometer" building kit contains several 2D polygons. Alexander, a geometry student, used the kit to build a 3D convex polyhedron. Next, Alexander disassembled the polyhedron and divided the polygons into two groups. Can it be possible that all polygons of each group can be assembled to a convex polyhedron so that each polygon from a given group is a face of the corresponding polyhedron and each of its faces is a polygon from this group?